HOW TO SOLVE WORD PROBLEMS

IAN SAMMIS

1. What is a word problem?

A *word problem* is basically a problem that attempts to mimic the way mathematics occurs in everyday life. Normally, you're asked in a math class to solve computational problems like this:

Problem 1. *What is* $50 - 50 \times .4$*?*

In a word problem, you might instead be given something more like this:

Problem 2. *Marcy is shopping for electronics. She sees a portable CD player that normally sells for $50 on sale for 40% off. What would she have to pay to buy the CD player?*

With a bit of looking, you will see that Problem 1 and Problem 2 are identical in the computation that they're asking you to carry out. This does *not* mean that the problems themselves are identical!

Problem 1 is asking for a pure computation. There's not much to show here, other than to show that you understand the order of operations involved:

$$\begin{aligned} 50 - 50 \times .4 &= 50 - 20 \\ &= \boxed{30} \end{aligned}$$

(Notice that we have drawn a box around our final answer. This is always a good practice, particularly if you plan to check your answer as a final step.)

Problem 2 is asking you a problem about what in principle at least is a real-world situation. This means that the solution shown above would *not* be adequate as an answer. Instead, you'd want to write something more like this:

Solution to Problem 2. Marcy is shopping for electronics. She sees a portable CD player that normally sells for $50 on sale for 40% off. What would she have to pay to buy the CD player?

Let P be the original sales price of the CD player. $P = \$50$.

Let s be the sales discount. $s = 40\% = 0.4$.

Let S be the price of the player on sale. We are asked to find S.

$$\begin{aligned} S &= P - Ps \\ S &= (\$50) - (\$50)(0.4) \\ S &= \$50 - \$20 \\ S &= \$30 \end{aligned}$$

$\boxed{\text{Marcy would have to pay \$30 to purchase the CD player on sale.}}$ □

A few things to notice:

- The solution to the word problem is much longer.
- The solution to the purely computation problem is actually sitting right in the middle of the word problem!
- The problem begins and ends in English. In particular, the answer to the English-language question that was asked is answered in English.
- The problem uses variables, and begins with entirely variables used.

Why take such care? Why not just write something very like the answer to Problem 1? The answer lies in the fact that as you move on to more and more advanced mathematics, and to more interesting problems in mathematical fields like physics, chemistry, and computer science, the problems will become more intricate, and less easy to solve "in passing." Being somewhat formal in your solutions will make it possible to answer even extremely fiddly questions.

2. How to approach a problem like this?

Let's take a look at a somewhat more fiddly question than the simple percentage sale that we examined in the last section.

Problem 3. *A washer and a dryer cost $450 combined. The cost of the washer is two times the cost of the dryer. What is the cost of the dryer?*

2.1. Rewrite the problem.

Although this seems like a waste of time, it's generally a good idea to make your written solution a complete description of how to solve the problem. This requires that the original problem—the one you're actually solving— be on your paper, not just in your textbook. This will also make it obvious what happened if you misread a number from the problem; otherwise misreads are very, very difficult for your grader to understand, and might be mistaken for more fundamental problems.

2.2. Declare your variables.

We now have to move from English to a more mathematical description of the problem. To do so, we need to define variables to represent the numbers of the problem. There's a very formulaic way to write this sort of thing: you write a sentence that looks like this:

Let *variable name* be *whatever the variable name represents.*

So, for example, you might write:

Let x be the cost of the dryer.

There's a bit of a stylistic problem here, though. The letter x, although traditional inside mathematics, doesn't really mean very much, or give you much of a hit of what it means. You've already partially solved this problem just by writing your "Let" sentence—you can always go back to the beginning of the problem and look up what x means—but it's usually safer to use a variable name that gives you a hint as to what it means. Thus:

Let d be the cost of the dryer.
Let w be the cost of the washer.

2.3. Optional: Draw a picture.

If your problem has any geometry to it—if you have things moving about, or some shaped pieces—then you should draw a picture of the problem next to your variable declarations. In this problem, that won't help much.

2.4. **Write the problem down mathematically.** Now that you have variables representing the numbers you don't know, you can translate your problem out of the English language. You can usually literally go one sentence at a time:

"A washer and a dryer cost \$450 combined":

$$w + d = \$450$$

"The cost of the washer is two times the cost of the dryer":

$$w = 2d$$

2.5. **Get rid of extra variables by substitution.** The problem now is that you have two variables, and really only know how to solve problems with one. There's a basic trick called substitution that can help.

Definition 1 (Substitution)**.** If you know that two things are equal, you can replace either of the things with the other, as long as you use parentheses correctly.

So, for example, we know that $w = 2d$. This means that anywhere we see a w we can replace it with $(2d)$:

$$\begin{aligned} w + d &= \$450 \\ (2d) + d &= \$450 \end{aligned}$$

In this case the parentheses were unnecessary, and so may be dropped:

$$2d + d = \$450$$

2.6. **DO THE MATH.** At this point, if all has gone well, you now have a math problem that basically isn't a word problem at all anymore. At this point, you can briefly settle down, forget you were ever working on a word problem, and do math:

$$\begin{aligned} 2d + d &= \$450 & \\ 3d &= \$450 & \text{combining like terms} \\ \frac{3d}{3} &= \frac{\$450}{3} & \text{dividing both sides by 3} \\ d &= \$150 & \text{doing the division} \end{aligned}$$

2.7. **Back–substitution and more math.** At this point you know one of your variables, d. To get the other one, go back to the equation you used for substitution earlier, and substitute in what d is.

$$\begin{aligned} w &= 2d \\ w &= 2(\$150) \\ w &= \$300 \end{aligned}$$

2.8. **Check.** Now that we have an answer to the mathematics, we can check. There are two types of checks to do:

(1) Mathematical checks: Make sure the answer fits the math that you were given. Is the washer twice the cost of the dryer, as claimed? Do the two together add up to \$450?

(2) Sanity checks: Is the answer obviously insane? Are the costs both positive numbers? Is each less than the total cost that both are supposed to be together? It's *much* worse to write down an insane answer than a merely wrong one. Giving a wrong (but plausible) answer means that you made a mistake somewhere. Giving an obviously crazy answer is more insulting—it means that you didn't think about your answer at all.

In this case, our answers meet both kinds of checks. Note that for the check step you don't need to actually write anything down unless you're specifically directed to do so by the problem or by your instructor.

2.9. **Answer the English-language question that you were asked!** We've now done all the necessary mathematics. What we haven't done is actually answered the question that was asked in the first place: What is the cost of the dryer?

$$\boxed{\text{The dryer costs \$150.}}$$

This answer does not, and should not, refer to any variables that you declared (it can refer to variables that were part of the statement of the problem, but shouldn't to variables that you invented yourself). It uses correct units: you don't claim that the dryer costs 150. (150 what? Half-pennies?) Most importantly, it is the English-language response to the English-language question that was asked by the problem. That was the goal.